波-流共同作用下的三维悬沙输运数学模型*

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摘要 依据三维动量方程和连续方程,通过将流场和悬沙场分别分解成3种不同时间尺度的速度和悬沙浓度的叠加,从理论上较严密地建立了适合于河口海岸地区、能合理反映波浪影响的三维流场方程和波-流共同作用下的三维悬沙扩散方程,从而构成了研究波-流共同作用下三维悬沙输运的数学模型,为进一步研究波-流共同作用下的悬沙输运特性提供了理论框架.

关键词 河口海岸 波-流共同作用 三维模型 泥沙输运

波浪、水流、泥沙3者之间的相互作用及影响是发生在河口海岸地区的普遍现象,因而探讨在波-流共同作用下的泥沙输运越来越引起国内外学者的重视.然而迄今为止,波-流作用下泥沙输运的计算大部分都是采用二维模型[1~8].为描述泥沙输运的三维特征,曹祖德等[9]采用分层二维模式,模拟三维波-流作用下的泥沙输运及海底演变.我们[10]从理论上给出了能清晰反映波、流共同作用下的三维悬沙扩散方程.O'Connor等[11],Katopodi等[12]和Lou等[13]分别进行波-流作用下的三维悬沙输运数值模拟.但他们采用的流场模型并不考虑波浪作用,而仅在扩散方程的涡动系数中考虑波浪影响.事实上,若能够在驱动泥沙输运的流场模型中引入波浪影响,则无疑在概念上将更加合理.特别对于河口海岸浅水区域,波浪对流场的影响有时是较明显的.

本文拟在已有研究成果的基础上,首先从理论上较为合理地导出波浪影响后的三维流场方程,然后给出更为一般的波-流共同作用下的三维悬沙扩散方程,为人们合理地研究波-流共同作用下的三维悬沙输运特性提供理论模型.

1 波浪影响下的三维流场方程

忽略分子粘性力,均质不可压流体三维基本方程可以写成

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+\frac{\partial u^2}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}-fv=-\frac{1}{\rho}\frac{\partial p}{\partial x}, \tag{2}$$

$$\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial v^2}{\partial y}+\frac{\partial vw}{\partial z}+fu=-\frac{1}{\rho}\frac{\partial p}{\partial y}, \tag{3}$$

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$$\frac{\partial w}{\partial t}+\frac{\partial uw}{\partial x}+\frac{\partial vw}{\partial y}+\frac{\partial w^2}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}-g, \tag{4}$$

式中 f 是柯氏参数. 为引入波浪对流场的影响,可将速度场分解成

$$\begin{aligned} u &= \bar{U}+u_w+u', \\ v &= \bar{V}+v_w+v', \\ w &= \bar{W}+w_w+w', \end{aligned} \tag{5}$$

式中假定 $\boldsymbol{V}=(\bar{U},\ \bar{V},\ \bar{W})$ 是大尺度的背景流场(如潮流、径流等), $\boldsymbol{v}_w=(u_w,\ v_w,\ w_w)$ 是波动质点速度, $\boldsymbol{v}'=(u',\ v',\ w')$ 是湍流脉动速度. 将(5)式代入(1)~(4)式,并用远大于湍流特征时间尺度的波动周期进行时间平均,经一系列演算后可得

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}+\frac{\partial W}{\partial z}=0, \tag{6}$$

$$\begin{aligned} &\frac{\partial U}{\partial t}+\frac{\partial U^2}{\partial x}+\frac{\partial UV}{\partial y}+\frac{\partial UW}{\partial z}-fV+\frac{\partial}{\partial x}(\overline{u_w^2}+\bar{u}_w^2)+\frac{\partial}{\partial y}(\overline{u_wv_w}-\bar{u}_w\bar{v}_w)+ \\ &\frac{\partial}{\partial z}(\overline{u_ww_w}-\bar{u}_w\bar{w}_w)+\frac{\partial}{\partial x}\overline{u'^2}+\frac{\partial}{\partial y}\overline{u'v'}+\frac{\partial}{\partial z}\overline{u'w'}=-\frac{1}{\rho}\frac{\partial\bar{p}}{\partial x}, \end{aligned} \tag{7}$$

$$\begin{aligned} &\frac{\partial V}{\partial t}+\frac{\partial UV}{\partial x}+\frac{\partial V^2}{\partial y}+\frac{\partial VW}{\partial z}+fU+\frac{\partial}{\partial x}(\overline{u_wv_w}-\bar{u}_w\bar{v}_w)+\frac{\partial}{\partial y}(\overline{v_w^2}-\bar{v}_w^2)+ \\ &\frac{\partial}{\partial z}(\overline{v_ww_w}-\bar{v}_w\bar{w}_w)+\frac{\partial}{\partial x}\overline{u'v'}+\frac{\partial}{\partial y}\overline{v'^2}+\frac{\partial}{\partial z}\overline{v'w'}=-\frac{1}{\rho}\frac{\partial\bar{p}}{\partial y}, \end{aligned} \tag{8}$$

$$\frac{\partial W}{\partial t}+\frac{\partial UW}{\partial x}+\frac{\partial VW}{\partial y}+\frac{\partial W^2}{\partial z}+\frac{\partial}{\partial z}(\overline{W_w^2}-\bar{W}_w^2)+\frac{\partial}{\partial z}\overline{W'^2}=-\frac{1}{\rho}\frac{\partial\bar{p}}{\partial z}-g, \tag{9}$$

式中 $U=\bar{U}+\bar{u}_w,\ V=\bar{V}+\bar{v}_w,\ W=\bar{W}+\bar{w}_w$,表示大尺度背景流与波动产生流动的叠加. 在导出(9)式时假定,波动及脉动的物理量在波周期内的平均值在水平方向的变化比垂直方向的变化要小得多. 特别地,若认为波动质点速度具有周期性,即取 $\bar{u}_w=\bar{v}_w=\bar{w}_w=0$,则(6)~(9)式可简化为

$$\frac{\partial\bar{U}}{\partial x}+\frac{\partial\bar{V}}{\partial y}+\frac{\partial\bar{W}}{\partial z}=0, \tag{10}$$

$$\begin{aligned} &\frac{\partial\bar{U}}{\partial t}+\frac{\partial\bar{U}^2}{\partial x}+\frac{\partial\bar{U}\bar{V}}{\partial y}+\frac{\partial\bar{U}\bar{W}}{\partial z}-f\bar{V}+\frac{\partial}{\partial x}\overline{u_w^2}+\frac{\partial}{\partial y}\overline{u_wv_w}+\frac{\partial}{\partial z}\overline{u_ww_w}+ \\ &\frac{\partial}{\partial x}\overline{u'^2}+\frac{\partial}{\partial y}\overline{u'v'}+\frac{\partial}{\partial z}\overline{u'w'}=-\frac{1}{\rho}\frac{\partial\bar{p}}{\partial x}, \end{aligned} \tag{11}$$

$$\frac{\partial \bar{V}}{\partial t}+\frac{\partial \bar{U}\bar{V}}{\partial x}+\frac{\partial \bar{V}^2}{\partial y}+\frac{\partial \bar{V}\bar{W}}{\partial z}+f\bar{U}+\frac{\partial}{\partial x}\overline{u_w v_w}+\frac{\partial}{\partial y}\overline{v_w^2}+\frac{\partial}{\partial z}\overline{v_w w_w}+$$

$$\frac{\partial}{\partial x}\overline{u'v'}+\frac{\partial}{\partial y}\overline{v'^2}+\frac{\partial}{\partial z}\overline{v'w'}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial y}, \tag{12}$$

$$\frac{\partial \bar{W}}{\partial t}+\frac{\partial \bar{U}\bar{W}}{\partial x}+\frac{\partial \bar{V}\bar{W}}{\partial y}+\frac{\partial \bar{W}^2}{\partial z}+\frac{\partial}{\partial z}\overline{w_w^2}+\frac{\partial}{\partial z}\overline{w'^2}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x}-g, \tag{13}$$

对于浅水区域，一般可取 $\left|\frac{\mathrm{d}\bar{W}}{\mathrm{d}t}\right| \ll \left|1-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial z}\right|$，于是，(13)式可进一步简化为

$$\frac{\partial}{\partial z}\overline{w_w^2}+\frac{\partial}{\partial z}\overline{w'^2}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial z}-g. \tag{14}$$

对上式垂向积分后可得

$$\bar{p}=\rho g(\bar{\eta}-z)-\rho\overline{w_w^2}-\rho\overline{w'^2}. \tag{15}$$

将(15)式代入(11)~(12)式，可得到适合于浅水区域波浪影响后的三维流场方程

$$\frac{\partial \bar{U}}{\partial t}+\frac{\partial \bar{U}^2}{\partial x}+\frac{\partial \bar{U}\bar{V}}{\partial y}+\frac{\partial \bar{U}\bar{W}}{\partial z}-f\bar{V}=-g\frac{\partial \bar{\eta}}{\partial x}+$$

$$\left(\frac{\partial M_{xx}}{\partial x}+\frac{\partial M_{yx}}{\partial y}+\frac{\partial M_{zx}}{\partial z}\right)+\left(\frac{\partial R_{xx}}{\partial x}+\frac{\partial R_{yx}}{\partial y}+\frac{\partial R_{zx}}{\partial z}\right), \tag{16}$$

$$\frac{\partial \bar{V}}{\partial t}+\frac{\partial \bar{U}\bar{V}}{\partial x}+\frac{\partial \bar{V}^2}{\partial y}+\frac{\partial \bar{V}\bar{W}}{\partial z}+f\bar{U}=-g\frac{\partial \bar{\eta}}{\partial y}+$$

$$\left(\frac{\partial M_{xy}}{\partial x}+\frac{\partial M_{yy}}{\partial y}+\frac{\partial M_{zy}}{\partial z}\right)+\left(\frac{\partial R_{xy}}{\partial x}+\frac{\partial R_{yy}}{\partial y}+\frac{\partial R_{zy}}{\partial z}\right), \tag{17}$$

式中 $M_{xx}=-(\overline{u_w^2}-\overline{w_w^2})$，$M_{xy}=M_{yx}=-\overline{u_w v_w}$，$M_{zx}=-\overline{u_w w_w}$，$M_{zy}=-\overline{v_w w_w}$，$M_{yy}=-(\overline{v_w^2}-\overline{w_w^2})$ 是由波动引起的动量通量；由脉动引起的 Reynolds 应力分别定义为 $R_{xx}=-(\overline{u'^2}-\overline{w'^2})$，$R_{xy}=R_{yx}=-\overline{u'v'}$，$R_{zx}=-\overline{u'w'}$，$R_{zy}=-\overline{v'w'}$，$R_{yy}=-(\overline{v'^2}-\overline{w'^2})$. 一般地，将 Reynolds 应力参数化，便可把波浪影响下的三维浅水流场方程表示成

$$\frac{\partial \bar{U}}{\partial t}+\frac{\partial \bar{U}^2}{\partial x}+\frac{\partial \bar{U}\bar{V}}{\partial y}+\frac{\partial \bar{U}\bar{W}}{\partial z}-f\bar{V}$$

$$=-g\frac{\partial \bar{\eta}}{\partial x}+\left(\frac{\partial M_{xx}}{\partial x}+\frac{\partial M_{yx}}{\partial y}+\frac{\partial M_{zx}}{\partial z}\right)+\frac{\partial}{\partial x}\left(A_x\frac{\partial \bar{U}}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_y\frac{\partial \bar{U}}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_z\frac{\partial \bar{U}}{\partial z}\right), \tag{18}$$

$$\frac{\partial \bar{V}}{\partial t}+\frac{\partial \bar{U}\bar{V}}{\partial x}+\frac{\partial \bar{V}^2}{\partial y}+\frac{\partial \bar{V}\bar{W}}{\partial z}+f\bar{U}$$

$$= -g\frac{\partial\bar{\eta}}{\partial y} + \left(\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} + \frac{\partial M_{zy}}{\partial z}\right) + \frac{\partial}{\partial x}\left(A_x\frac{\partial\bar{V}}{\partial x}\right) + \frac{\partial}{\partial y}\left(A_y\frac{\partial\bar{V}}{\partial y}\right) + \frac{\partial}{\partial z}\left(A_z\frac{\partial\bar{V}}{\partial z}\right), \tag{19}$$

式中 A_x，A_y，A_z 为湍流粘性系数.

海面及海底的运动学边界条件为

$$\begin{aligned}\bar{W} &= \frac{\partial\bar{\eta}}{\partial t} + \bar{U}\frac{\partial\bar{\eta}}{\partial x} + \bar{V}\frac{\partial\bar{\eta}}{\partial y}, \quad z = \bar{\eta}(x,y,t),\\ \bar{W} &= -\left(\bar{U}\frac{\partial h}{\partial x} + \bar{V}\frac{\partial h}{\partial y}\right), \quad z = -h(x,y).\end{aligned} \tag{20}$$

依据波动理论或实际计算得到三维波动解,然后合理确定湍流粘性系数,联立(10)、(18)～(20)式就可求得波动影响后的三维流场. 若背景流场为潮流,且不考虑波动对流场的影响,即在(5)式中取 $v_w = 0$,则波动引起的动量通量为零. 此时(18)～(19)式可简化为人们熟知的三维浅水潮流方程:

$$\frac{\partial\bar{U}}{\partial t} + \frac{\partial\bar{U}^2}{\partial x} + \frac{\partial\bar{U}\bar{V}}{\partial y} + \frac{\partial\bar{U}\bar{W}}{\partial z} - f\bar{V}$$

$$= -g\frac{\partial\bar{\eta}}{\partial x} + \frac{\partial}{\partial x}\left(A_x\frac{\partial\bar{U}}{\partial x}\right) + \frac{\partial}{\partial y}\left(A_y\frac{\partial\bar{U}}{\partial y}\right) + \frac{\partial}{\partial z}\left(A_z\frac{\partial\bar{U}}{\partial z}\right), \tag{21}$$

$$\frac{\partial\bar{V}}{\partial t} + \frac{\partial\bar{U}\bar{V}}{\partial x} + \frac{\partial\bar{V}^2}{\partial y} + \frac{\partial\bar{V}\bar{W}}{\partial z} + f\bar{U}$$

$$= -g\frac{\partial\bar{\eta}}{\partial y} + \frac{\partial}{\partial x}\left(A_x\frac{\partial\bar{V}}{\partial x}\right) + \frac{\partial}{\partial y}\left(A_y\frac{\partial\bar{V}}{\partial y}\right) + \frac{\partial}{\partial z}\left(A_z\frac{\partial\bar{V}}{\partial z}\right). \tag{22}$$

2 波-流共同作用下的悬沙扩散方程

据质量守恒定律,得到三维悬沙扩散方程为

$$\frac{\partial c}{\partial t} + \frac{\partial uc}{\partial x} + \frac{\partial vc}{\partial y} + \frac{\partial wc}{\partial z} = \frac{\partial\omega_s c}{\partial z}, \tag{23}$$

式中 c 为悬沙浓度,ω_s 为悬沙沉降速度. 类似于速度场,把悬沙浓度场也分解成由背景流、波动及湍流脉动引起的 3 部分,即

$$c = \bar{C} + c_w + c', \tag{24}$$

将(5),(24)式代入(23)式,然后求波周期平均,可得在波-流共同作用下的三维悬沙扩散方程的一般形式为

$$\frac{\partial C}{\partial t} + \frac{\partial UC}{\partial x} + \frac{\partial VC}{\partial y} + \frac{\partial WC}{\partial z} - \frac{\partial\omega_s C}{\partial z}$$

$$= \frac{\partial\tau_{Rx}}{\partial x} + \frac{\partial\tau_{Ry}}{\partial y} + \frac{\partial\tau_{Rz}}{\partial z} + \frac{\partial\tau_{wx}}{\partial x} + \frac{\partial\tau_{wy}}{\partial y} + \frac{\partial\tau_{wz}}{\partial z}, \tag{25}$$

式中 $C = \bar{C} + \overline{c_w}$，$U$，$V$，$W$ 是波动影响下的三维流场速度,

$$\tau_{Rx} = -\overline{u'c'_s},\ \tau_{Ry} = -\overline{v'c'_s},\ \tau_{Rz} = -\overline{w'c'_s},$$

$$\tau_{wx} = -(\overline{u_w c_w} - \overline{u_w}\,\overline{c_w}),\ \tau_{wy} = -(\overline{v_w c_w} - \overline{v_w}\,\overline{c_w}),\ \tau_{wz} = -(\overline{w_w c_{sw}} - \overline{w_w}\,\overline{c_w})$$

分别表示由湍流脉动及波动引起的悬沙扩散.

若假定波动质点速度及由波动引起的悬沙浓度具有周期性,并分别将脉动和波动引起的悬沙扩散项参数化,即取

$$\tau_{Rx} = \varepsilon_{cx}\frac{\partial\bar{C}}{\partial x},\ \tau_{Ry} = \varepsilon_{cy}\frac{\partial\bar{C}}{\partial y},\ \tau_{Rz} = \varepsilon_{cz}\frac{\partial\bar{C}}{\partial z}, \tag{26}$$

$$\tau_{wx} = \varepsilon_{wx}\frac{\partial\bar{C}}{\partial x},\ \tau_{wy} = \varepsilon_{wy}\frac{\partial\bar{C}}{\partial y},\ \tau_{wz} = \varepsilon_{wz}\frac{\partial\bar{C}}{\partial z}. \tag{27}$$

将(26)式和(27)式代入(25)式,可得到波-流共同作用下的三维悬沙扩散方程为:

$$\frac{\partial\bar{C}}{\partial t} + \frac{\partial\bar{U}\bar{C}}{\partial x} + \frac{\partial\bar{V}\bar{C}}{\partial y} + \frac{\partial\bar{W}\bar{C}}{\partial z} - \frac{\partial\omega_s\bar{C}}{\partial z} = \frac{\partial}{\partial x}\left(\varepsilon_x\frac{\partial\bar{C}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\varepsilon_y\frac{\partial\bar{C}}{\partial y}\right) + \frac{\partial}{\partial z}\left(\varepsilon_z\frac{\partial\bar{C}}{\partial z}\right). \tag{28}$$

式中 $\varepsilon_x = \varepsilon_{cx} + \varepsilon_{wx}$, $\varepsilon_y = \varepsilon_{cy} + \varepsilon_{wy}$, $\varepsilon_z = \varepsilon_{cz} + \varepsilon_{wz}$表示由脉动和波动共同引起的悬沙扩散系数, $\bar{U}$, $\bar{V}$, $\bar{W}$ 是波浪影响下三维流场速度,可据上面给出的波浪影响下的三维浅水流场方程确定. 我们曾取背景流场 $\boldsymbol{V} = (\bar{U}, \bar{V}, 0)$,导出波-流共同作用下的三维悬沙扩散方程[10]. 此处保留背景流场的垂向速度,给出了更一般的波-流共同作用下的三维悬沙扩散方程. 显然,若不考虑波动对悬沙的影响,则(28)式简化为人们熟知且常使用的仅在水流作用下的三维悬沙扩散方程

$$\frac{\partial\bar{C}}{\partial t} + \frac{\partial\bar{U}\bar{C}}{\partial x} + \frac{\partial\bar{V}\bar{C}}{\partial y} + \frac{\partial\bar{W}\bar{C}}{\partial z} - \frac{\partial\omega_s\bar{C}}{\partial z} = \frac{\partial}{\partial x}\left(\varepsilon_{cx}\frac{\partial\bar{C}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\varepsilon_{cy}\frac{\partial\bar{C}}{\partial y}\right) + \frac{\partial}{\partial z}\left(\varepsilon_{cz}\frac{\partial\bar{C}}{\partial z}\right). \tag{29}$$

式中速度场依据(10)、(20)~(22)式确定.

3 结语

本文从最基本的 N-S 方程出发,导出了波动影响下的三维浅水流场基本方程,以及在波-流共同作用下的三维悬沙扩散方程. 这些方程的导出,有助于人们进一步研究河口海岸地区波动影响下的流场及悬沙场的三维结构,探讨波-流共同作用下的三维悬沙输运特性. 此外,文中给出的波-流共同作用下三维悬沙扩散方程的形式具有普遍性,同样适用于河口海岸地区盐度场等在波-流共同作用下扩散过程的研究. 尽管在导出波浪影响下的三维流场方程时,取密度为常数,但采用本文给出的推导方法,也不难导出波浪影响下的三维斜压浅水流场方程. 但尚需指出的是,本文研究只是从理论上建立模型,若将其应用于实际,必须作进一步理论分析,将波浪引起的动量通量参数化,并通过与现场观测和室内实验相结合,研究波、流特性以及在波-流共同作用下的泥沙特性,以便确定模型中的有关参数. 这些问题我们正在继续研究,部分成果将另文发表.

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